What is coherence?

The coherence function (also the magnitude squared coherence), $\gamma_{xy}(f)$, between two signals $x(t)$ and $y(t)$, is defined as:

$$\gamma_{xy}(f) = \frac{|R_{xy}(f)|^2}{R_x(f)R_y(f)}$$

where $R_{xy}(f)$ is the cross-spectral density and $R_x(f)$ and $R_y(f)$ are the spectral densities. The coherence is the proportion of power of $x(t)$ at frequency $f$ that can be explained by a linear filter transformation of $y(t)$ at frequency $f$. Since a linear filter cannot change the frequency of a component, it can only damp or amplify different frequency components. In this paper the coherence is allowed to vary with time. We denote the time-local coherence, $\gamma_{xy}(t,f)$. Two straightforward methods to estimate time-local coherence are described and compared.

Coherence estimation using Multiple Windows

The key to estimate time-local coherence is to have many uncorrelated time-frequency estimates at approximately the same time and frequency. This can be achieved by computing short-time Fourier transforms (STFT) using multiple windows (MW). The windows are designed to give low variance, even though they overlap within a single frame. The following set of multiple windows are evaluated:

- Welch multiple windows (WinR)
- Peak matched multiple windows (PM MW)

These windows are designed to give small correlation between sub-spectra when the spectrum of the random process includes peaks and notches, i.e., spectra with large dynamics.

Locally stationary processes: optimal windows (GOPT MW). These windows are mean square error optimal for a locally stationary process.

How to evaluate coherence estimation methods?

Two test signals $x(t)$ and $y(t)$, made to offer the same challenge as real EEG signals, are simulated. The signals have one known component in common, located at $t = 0.5$ and $f = 15$ Hz. They are surrounded by noise at $t = 15 \pm 3$ Hz and spikes, schematically illustrated below.

<table>
<thead>
<tr>
<th>x in FP-plane</th>
<th>y in TFP-plane</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 Hz</td>
<td>15 Hz</td>
</tr>
<tr>
<td>Spike</td>
<td>Alpha-process</td>
</tr>
<tr>
<td>White noise</td>
<td>Alpha-process</td>
</tr>
<tr>
<td>common to both x and y</td>
<td></td>
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</tbody>
</table>

We expect the coherence of $y(t)$ and $y(t)$ to be zero everywhere except around $t = 0$ and $f = 15$ Hz. The signal-to-noise ratio (SNR) is defined as the mean estimated coherence in the area divided by the mean in a larger surrounding area. By simulating pairs of $x(t)$ and $y(t)$, we can get a picture of different coherence estimation methods' SNR distribution.

Coherence estimation on real EEG signals

The electroencephalogram technique (EEG) visualizes the brain's activity in a low-cost and in a simple and non-invasive way. The spatially averaged activity of the cortical areas interacts with performing various cognitive tasks. It has been shown that when a particular task is carried out, the involved cortical areas start synchronized activity within specific frequency bands.

EEG was recorded from a subject presented to 9 Hz flickering light. Coherence between channels P3 and P4 were estimated using four different methods. As seen in the figure, the coherence at 9 Hz is high when the light is flickering. However, there is also high coherence in some other time frequency areas.